Theories as Practices

A Use-Based Account of Interpretation and Equivalence

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Abstract

This paper develops a framework that bridges the gap between two prominent approaches to theory interpretation and theoretical equivalence: content-based and formal. Content-based approaches treat the process of interpretation as one of assigning semantic objects—like propositions or truthmakers—to sentences of a language. Formal approaches, by contrast, treat interpretation as the process of delineating a theory's semantic architecture. Both face difficulties. Content-based views risk trivializing equivalence, while formal approaches struggle with cases where formally identical theories diverge in meaning. In response, I develop a metasemantic account putting community use at the center of the picture. Although the central insight—that use determines meaning—is familiar, formalizing it enables progress, or at least a new point of view, on a wide range of issues. In particular, I show how the framework refines both formal and content-based criteria for theoretical equivalence, plants the seeds of an account of theoretical expansion and revision, and illuminates issues in semantics, logic, and (meta)metaphysics.

1 Introduction

Philosophers of science and metaphysicians alike often ask two questions concerning theories: "*When do two theories say the same thing about the world?*" and "*What commitments do our theories carry about how the world is?*" These questions are central to philosophical debate within these fields: they shape debates about scientific realism, ontology, the interpretation of physics, and even whether certain theoretical disputes are substantive or merely verbal.

Traditionally, two broad approaches to answering these questions have dominated. *Formal approaches* treat theories as mathematical structures—like sets of sentences in formal languages (Quine, 1975; Glymour, 1977; Barrett and Halvorson, 2016), collections of models (Van Fraassen, 1987), or categories (Weatherall, 2017)—and assess equivalence by logical, model-theoretic, or category-theoretic criteria. *Content-based approaches* focus instead on the "worldly" content of theories as construed in various metaphysical approaches to semantics, treating theories or their languages as mappings from sentences to *contents*—propositions (Stalnaker, 2007), truthmakers (Fine, 2017), possibilities (Holliday, 2021), and the like—, and assessing equivalence in terms of whether the theories or underlying languages are mapped to the same contents.

Both approaches have been argued by proponents of opposing views to face serious difficulties. Formal approaches, for instance, have been argued to face the problem of *formal underdetermination*, which is their failure to account for the fact that formally *identical* theories can represent different facts.

> Let the two theories be 'All lions have stripes', and 'All tigers have stripes', with all the words in both theories taking on their usual meanings. The theories are inter-translatable in the purely formal sense. They are exactly alike in logical form and one can be obtained from the other by a simple term for term substitution. But they are most assuredly not equivalent [...] mere commonality of logical form, even of a total theory when compared with another total theory, is certainly not by itself sufficient for theoretical equivalence. The meanings of the terms in the theories, however construed, are crucial to questions of equivalence. (Sklar, 1982, p. 98)

Although first noted by Sklar, this problem has been later revisited by Coffey (2014), Sider (2020), and Teitel (2021), in different ways. Faced with this problem, proponents of formal approaches often respond by saying that not *every* formal equivalence relation counts, but not much progress has been made in specifying which ones do. Perhaps we should require the formal equivalence relation to "respect the meanings" of the relevant theoretical terms, but it is unclear how to specify this in a way that is both precise and non-circular.[1] Perhaps we should require that formally equivalent theories also be "empirically equivalent", but it is not clear that we have an adequate account of *empirical equivalence* that works across the board to formulate a principled criterion.[2]

It is clear that this problem needs to be addressed. But certain ways of raising it have led to dead ends. For instance, Teitel (2021) frames the difficulty in terms of what he calls *trivial semantic conventionality*: any representational vehicle, he argues, can in principle be used to represent the world as being any way whatsoever (p. 4125). On this basis, he concludes that no purely formal

[1] See Sider (2020, §5.1) for a more detailed discussion of this problem.
[2] See (Teitel, 2021, p. 4129) for a more detailed discussion of this problem.

relation between representational vehicles can suffice for theoretical equivalence. I agree that formal underdetermination is real—even if trivial semantic conventionality, in Teitel's sense, is false. Even granting that there are constraints on how representational vehicles *can* be used, it remains possible for formally identical systems to differ radically in what they say about the world. The challenge is to explain this difference in a principled way.

From the opposite direction, Dewar (2023; 2024) argues that Teitel's diagnosis itself rests on an overly permissive conception of interpretation—what I will call the "sticker-book picture"[3]—, according to which interpreted theories are simply uninterpreted formalisms plus a mapping from their sentences to contents. This, he contends, *overgenerates* interpreted theories—since any mapping from vehicles to contents would qualify as one. Thus, he ultimately rejects the sticker-book picture, and trivial semantic conventionality as well. Additionally, he argues that this picture *reverses the direction of explanation* by starting with uninterpreted formalisms on one side and meanings on the other, while in practice we come to grasp the content of a theory through its application and articulation, and not the other way around.

I agree with Dewar that an account of equivalence must reject this sticker-book conception and explain, at least in outline, how theories acquire their content. But it is not clear that Teitel, or any proponent of content-based approaches, endorses this picture of interpretation. Also, even if they do, it is not clear that rejecting the sticker-book picture alone dissolves the problem of formal underdetermination: even once we restrict which mappings count as legitimate, it remains the case that formally identical theories can (and do) radically differ in content. But, at the very least, Dewar's critique highlights the need for a *positive* account of interpreted theories that goes beyond the sticker-book picture, and which is not vulnerable to the overgeneration or direction-of-explanation objections. So far, no such account has been proposed by either side of the debate.

What is needed, then, is a positive account of interpreted theories that goes beyond the sticker-book picture while addressing the problems raised by both sides: overgeneration, direction-of-explanation, and formal underdetermination. Such an account must explain how theories acquire their content in a way that constrains interpretation without treating meaning as something fixed independently of use.

The present paper is an attempt to provide such a positive account of interpreted theories. On the view I propose, interpreted theories are not mappings to semantic objects, nor bare formal structures (plus "empirical content"). Rather, they are constituted by *patterns of use*—practices that constrain interpretation. Relatedly, the process of *interpreting* a theory or language should be seen as a process of *using* its syntax in a way that "filters out" certain mappings from sentences to contents. The only remaining, viable mappings are those consistent with how we actually deploy the theory. If all these viable mappings align on a particular content for a given sentence or expression, that

[3] I am borrowing the sticker-book analogy from Dewar (2024), who is in turn borrowing it from Price (2011).

content “comes into focus” through the lens of usage. Hence, I will call this the *filtering lens picture of interpretation*. Because this picture embeds use directly in the theory’s formal articulation, it bridges the gap between a theory’s formal structure and its content through *representational use*. This in turn allows progress, or at least a new point of view, on several fronts: it refines equivalence criteria, clarifies theory expansion and revision, supports alternative accounts of quantification, and sheds light on debates in metametaphysics and structural realism.

The paper proceeds as follows. In §2, I introduce and motivate the filtering lens picture and explain how it avoids the three problems mentioned above. In §3, I provide a precise formal articulation of this picture, making explicit the conceptual relations between community use, interpretative constraints, and content. In §4, I apply the framework to a variety of problems concerning the structure of theories, like theoretical equivalence, theoretical expansion and integration, and theory revision in the philosophy of science. These applications shed light on broader positions relating science to metaphysics, like ontic structural realism. In §5, I apply the framework to problems in semantics, logic, and metaphysics, like alternative accounts of quantification, and quantifier variance. I conclude in §6.

2 The Filtering Lens Picture

Recall that content-based approaches often treat interpreted theories as representational vehicles (e.g., sentences) plus a mapping from them to semantic contents (e.g., propositions, sets of worlds). This suggests a picture in which interpretation is a process of “matching” the former with the latter, like a child with a sticker-book. Although no philosopher explicitly endorses such an overly simplistic view of interpretation, it can sometimes be implicitly assumed if interpreted theories are treated simply as mappings from syntactic items to contents. On the other hand, while formalists occasionally acknowledge the importance of use, their actual treatments typically abstract away from it: they identify theories with mathematical structures and appeal to formal properties alone when assessing equivalence (Weatherall, 2016; Rosenstock, Barrett, and Weatherall, 2015; Barrett and Halvorson, 2016; Dewar, 2019).

Against these two, I want to propose a third approach which begins by bringing to the foreground a widely acknowledged but often underappreciated fact: that *use determines meaning*. The way I want to bring attention to this fact is by conceiving interpretation as a process of filtering the space of possible mappings from sentences to contents. This filtering process is guided by constraints on this space produced by facts about how we use the language. These facts are what I will call *use-facts*. This process is better understood through an example. Consider a language L syntactically resembling propositional logic, and let c be a community of users. Suppose that, in c’s practice, asserting $\ulcorner \phi \land \psi \urcorner$ thereby commits you to both ϕ and ψ. We can represent this use-fact as:

($U_{\wedge E}$) c uses L in such a way that, for any sentences ϕ and ψ, affirming $\ulcorner \phi \wedge \psi \urcorner$ commits you to both ϕ and ψ.

If we deploy possible-world semantics, an "interpretation" i assigns a set of worlds $i(\phi)$ to each sentence ϕ, which we should think of as the worlds where ϕ is true. To remain compatible with ($U_{\wedge E}$), this mapping or content assignment must be such that $i(\ulcorner \phi \wedge \psi \urcorner)$ is a subset of $i(\phi) \cap i(\psi)$. This rules out any interpretation i that fails to satisfy the following condition:

($C_{\wedge E}$) For any sentences ϕ and ψ, we have $i(\ulcorner \phi \wedge \psi \urcorner) \subseteq i(\phi) \cap i(\psi)$.

This is the *interpretative constraint* induced by the use-fact ($U_{\wedge E}$). Other facts about how c uses L will impose other constraints on content assignments. For instance, if c uses L such that anyone committed to both ϕ and $\ulcorner \neg\phi \urcorner$ is "out of bounds," then we would have another use-fact, which would impose another constraint on content assignments—plausibly, that $i(\phi) \cap i(\ulcorner \neg\phi \urcorner) = \varnothing$. Similar constraints connecting use and interpretation for other logical connectives can be found in Hlobil and Brandom (2025).

Notice that ($U_{\wedge E}$) and ($C_{\wedge E}$) concern the use and interpretation of the *logical* constant '$\wedge$'. But we can extend this idea to the "non-logical" vocabulary of L. For instance, we might focus on how expressions like 'is red' or 'is a chair' are used, and on the corresponding constraints on content assignments that these ways of use impose. Contrary to the case of '$\wedge$' and other "logical" vocabulary, the use-facts associated with these expressions involve more than just conditional commitments. They also involve facts connecting specific situations to "speakers" or "hearers" who utter, react, or otherwise interact with sentences containing the relevant expressions. Providing a comprehensive account of these facts for any natural language or even a relatively simple theory is a daunting task, and it is not clear whether it can be done in a principled way. But Skyrms (2010) provides an analysis of simplified cases of this kind for more primitive forms of signaling systems. For instance, cases are discussed where vervet monkeys (c_{vm}) use different calls in a signaling system (S) to signal different predators, giving rise to use-facts like the following (p. 23):

(U_1) c_{vm} uses S in such a way that its members call *bark* in the presence of a leopard.

(U_2) c_{vm} uses S in such a way that its members run up trees when they hear the call *bark*.

This constrains admissible content assignments—not by stipulating inferential roles, but by excluding content assignments incongruent with patterns of use. In this case, the constraint is that the content of *bark* must be a subset of the set of (centered) worlds where leopards are nearby (if the calls are interpreted descriptively) or of the set of worlds where the monkeys are running up trees (if they are interpreted as imperative commands).

Admittedly, the use-facts associated with any sufficiently complex language or theory are immensely complex, and it is perhaps not even possible to make them fully explicit. Even so, this does not undermine the core idea: that such use-facts exist, and that they place genuine constraints on interpretation. Indeed, many theorists have attempted to formalize aspects of this idea in different ways—Lewis (1975; 1984), Skyrms (2010), and Hlobil (2025) among them. Their approaches differ, but all share the core idea that semantic content is constrained by how a community uses its language. The filtering lens framework is compatible with all such approaches. Moreover, it does not assume that all use-facts must be made explicit, and shows how even partial information can produce interpretative constraints and guide content determination in a principled way.

Still, it is useful to imagine that we could, in principle, make all use-facts explicit and clearly articulate how they induce interpretative constraints. This is the ideal that the current picture aims to approximate. Now suppose, following this ideal, that we learn *all* the other facts like ($U_{\wedge E}$) about the community's use of L. Then we could characterize c's *way of using* L as given by the conjunction of all these use-facts. We can represent this by a sentence of the following form:

(U_{tot}) c uses L in way u.

This is in turn a use-fact, since the conjunction of all facts about how c uses L is itself a fact about how c uses L. Thus, it itself imposes a constraint on content assignments i of L if they are taken as compatible with c's understanding of L:

(C_{tot}) i satisfies condition $C(u)$.

Here, $C(u)$ is the condition that content assignments of L must satisfy in order to be compatible with the way of using L given by u.[4] Call this the *interpretative constraint* induced by the use-fact (U_{tot}). Note that the notion of "compatibility" here is metasemantic rather than logical: it concerns how community practices shape interpretative possibilities. As stated before, this relation is complex, but does not need to be *fully* explicitated in order to make progress.

Now, take a sentence ϕ of L and consider the question of what the content of ϕ is given that the community c uses L in way u. The current picture assigns P as the "determinate" content of ϕ whenever the following condition holds:[5]

[4]It is worth noting that I am not assuming u to be given by *every* instance of a community member using L. There might, of course, be "incorrect" uses of L. The problem of distinguishing between correct and incorrect uses is a complex one, and I will not address it here beyond saying that it is a problem that is not unique to the filtering lens picture. But following the spirit of this picture, I take it that a necessary condition for a use to be "correct" is that it be common enough to define a *pattern* among community members using L. See Lewis (1975) and Skyrms (2010) for more discussion on this point.

[5]That content is determined as a single set of worlds is a simplifying assumption. Throughout this paper, I will uphold this assumption, but it is worth noting that it is not necessary for the core idea of the filtering lens picture to hold.

(Δ) For any content assignment i of L satisfying $C(u)$, $i(\phi) = P$.

Thus, one can think of the content of ϕ as the content that "comes into focus" when we consider how content assignments that are incompatible with the community's use of L are "filtered out", which justifies the name of the "filtering lens" picture. Of course, the content of a sentence is only one of the features of a theory that can be determined by use-facts. More complex features, like *relations* between the contents of sentences, can also be determined by them, as shown by ($C_{\wedge E}$). It is not hard to see how to extend this idea to sets of sentences, or even models, as representational vehicles. In doing so, the core idea remains the same. This motivates the need for a more precise framework for representing these constraints. The next section introduces such a framework, which allows us to track how content assignments are shaped by use-facts across possibilities of use.

But before this, it is worth noting how this approach addresses the three problems mentioned in §1. First, we avoid the overgeneration problem because not all mappings from syntax to content are allowed, but only those that aren't filtered out by the community's use of the language. Second, we avoid the direction-of-explanation problem by starting with the actual community's usage and considering the interpretation of a sentence (or a set of sentences, or a model) to be *definite* only if it is determined by this usage. Third, we avoid the formal underdetermination problem: cases where formally identical theories have different contents are simply cases where the community's use of the representational system, and the corresponding constraints on content assignments, are different.

Finally, note that although the filtering lens picture allows formally identical theories to have different contents, it is not inherently hostile to formal approaches to equivalence. In particular, it doesn't support the conclusion that "formal criteria are of limited non-mathematical interest" (Teitel, 2021, p. 4120). Rather, we can naturally think of formal approaches as tacitly relying on use-facts like ($U_{\wedge E}$) and their associated constraints. This allows us to think of them as apt to be absorbed into more comprehensive frameworks: ones that bring in additional constraints which are *not* associated with the logical vocabulary. In the next sections, I will provide a formal articulation of the filtering lens picture, and show how it can be used to address a variety of problems in philosophy of science, semantics, and metaphysics, thereby showing that the picture is independently useful.

3 A Formal Interpretative Framework

I now turn to the formal presentation of the filtering lens picture. The aim of this section is to make the picture of §2 precise enough to be applied to questions about theoretical equivalence (§4.1), theory expansion and revision (§4.2, §4.3), and related issues in semantics and metaphysics (§5). The formalization is therefore a *tool* for deploying the picture, not a self-standing formal theory of equivalence. Without this metasemantic picture concerning communities, use-facts, and interpretative constraints, the framework below would amount to just another formal approach of the kind criticized in §1. The later sections (§4, §5), in turn, illustrate how the metasemantic picture guides the application of the formal machinery.

This order of presentation is deliberate. On the filtering lens picture, an interpreted theory is a system whose representational role is constrained by patterns of use. In presenting the formal framework only after introducing the metasemantic picture that constrains its application (§2), and then showing how the picture guides the application of the formal framework (§4, §5), I am following that very model: the formalism is not offered independently, but only under a specification of how it is to be used.

Now, turning to the formal framework, I will start by defining the main components of the filtering lens picture and then combine them into what I will call a *use-model*.

Worlds. We will consider a set W of "worlds" that will serve *both* to track how languages are used and to construct the contents of their sentences. One should think of each world $w \in W$ as (i) settling all facts about how the languages or representational systems of interest are used, and (ii) settling the facts that these languages or representational systems aim to represent.[6]

Languages. We will consider *uninterpreted languages* which will be elements of a certain domain D^ℓ. Each language is assumed to contain a set of *sentences*—elements of a domain D^s. We will need a relation $\in^\ell \subseteq D^s \times D^\ell$ specifying which sentences belong to which languages. For each language L, D^s_L will be the subset of D^s containing exactly the sentences of L.

Content Assignments. A *mapping* or *content assignment* will be a function from D^s to some set of contents D^t. Define the domain of mappings D^π as $D^s \to D^t$, the set of functions from D^s to D^t. Given a mapping i and a language L, we can obtain *i's restriction to* L (written '$i|_L$') by functionally restricting i to D^s_L, so the set D^π suffices to represent *all* possible content assignments of L. For simplicity, we can assume that D^t is the powerset of the set of worlds, $\mathcal{P}(W)$. If we want to extend this framework to account for context-sensitivity, we need to change D^t so that its elements can take a *context* as an argument, but I will omit this for simplicity.

Use-Facts and Communities. We will consider *communities* as elements of a certain domain D^κ. For simplicity, we can assume that each community exists in every world, and this be necessitists, but relaxing this assumption should not affect the core of the framework. We then consider a relation $U \subseteq W \times D^\kappa \times D^\ell$ specifying which languages are used by which communities in which worlds, where $\langle w, c, L\rangle \in U$ should be taken to hold whenever c uses L in w. For each triple $\langle w, c, L\rangle$

[6] In possible world semantics, it is normally assumed that possible worlds settle *all* the facts about how the world is. Since recent work in metaphysics has shown this condition to not be uncontroversial (see Bacon 2023, Ch.6), I will not assume it here.

in U, we have a nonempty set $\mathcal{F}(w, c, L) \subseteq \mathcal{P}(W)$ containing all the facts *about how* c *uses* L *in* w. These facts generate constraints on admissible content assignments. To ensure they adequately satisfy their role, we require that $\mathcal{F}(w, c, L)$ satisfy the following:

Facticity. For all $P \in \mathcal{F}(w, c, L)$, $w \in P$.

Adequacy. For all $P \in \mathcal{F}(w, c, L)$, $P \subseteq \{w' \in W \mid \langle w', c, L\rangle \in U\}$.

Closure. For all nonempty $\mathcal{S} \subseteq \mathcal{F}(w, c, L)$, the intersection $\bigcap \mathcal{S}$ is in $\mathcal{F}(w, c, L)$.

Intuitively, Facticity ensures that use-facts about how c uses L in w hold in w, Adequacy ensures that they require the usage of L by c, and Closure ensures that the (perhaps infinitary) conjunction of some facts about how c uses L in w is itself a fact about how c uses L in w.

There is an important question of how to structure $\mathcal{F}(w, c, L)$: in principle, $\mathcal{F}(w, c, L)$ could consist of *one* use-fact specifying the whole of c's use of L in w, or it could consist of many use-facts specifying different aspects of c's use of L in w. While the formal model is agnostic about this question, a helpful way to implement the filtering lens picture is to assume that for every basic expression ε of L, there is *at least one* use-fact $P_\varepsilon \in \mathcal{F}(w, c, L)$ specifying how c uses ε in w. This follows the lines of the examples provided in §2 and clarifies the *role of syntax* in the filtering lens picture: languages (or representational systems) are syntactically structured not because they attempt to "mirror" the structure of the world, but because of the *possibilities of use* this syntactic structure allows. The resulting picture of the role of syntax is thus substantially different than the one offered by many conceptions of how our theories represent (or ought to represent) the world (Sider, 2011).

Even here, however, there remains the question of what kind of facts we are dealing with. In the formal model, use-facts appear simply as sets of worlds subject to Facticity, Adequacy, and Closure constraints, which are linked to syntax through their role in constraining admissible content assignments. But this still leaves open a substantive metasemantic issue: whether such facts concern only the internal cognitive states of speakers, or also broader environmental relations among speakers, linguistic communities, and their surroundings. A *semantic internalist* will favor the former, while an *externalist* will favor the latter. The filtering lens picture is intended to be neutral on this point. Its commitments lie in how use-facts function—namely, as modally robust regularities that constrain which content assignments are admissible—not in their "metaphysical character". The hope is that the framework can be fruitfully adopted regardless of one's metasemantic commitments in this regard, and that it offers a way to connect questions of meaning and representation with the actual patterns of use that structure our linguistic and theoretical practices.

Interpretation. For each triple $\langle w, c, L\rangle$ in U, there will be a set of content assignments $C(w, c, L) \subseteq D^\pi$, thus producing a function $C : U \to \mathcal{P}(D^\pi)$. One should think of $C(w, c, L)$ as the set of all content assignments that are compatible with c's usage of L in w. As explained in §2, this notion of compatibility is *metasemantic*: it concerns how community practices shape interpretative possibilities. Since interpretation in this picture depends on the *use-facts* and not on any other facts, the interpretation function C needs to satisfy the following constraint:

Supervenience. For all $w_1, w_2 \in \bigcap \mathcal{F}(w, c, L)$, $C(w_1, c, L) = C(w_2, c, L)$.

This condition ensures that the interpretation of L in w is the same for all worlds that satisfy the same use-facts about c's use of L in w. Finally, we can extend C to a function on the whole of $W \times D^\kappa \times D^\ell$ by setting $C(w, c, L) = \varnothing$ whenever $\langle w, c, L \rangle \notin U$.

3.1 A Model of Use

We now have all the elements we need to define a *model* of the filtering lens picture, which I will call a *use-model*:

Use-Model. A *use-model* is a tuple $\mathcal{M} = \left\langle W, D^\ell, D^s, \in^\ell, D^\kappa, U, \mathcal{F}, C \right\rangle$ where:

- W is a nonempty set of *worlds*. We define $D^t := \mathcal{P}(W)$.
- D^ℓ is a nonempty domain of *uninterpreted languages*.
- D^s is a nonempty domain of *sentences*. We define $D^\pi := D^s \to D^t$.
- $\in^\ell \subseteq D^s \times D^\ell$ is a *belonging* relation, such that for all $\phi \in D^s$, there is at least one $L \in D^\ell$ with $\phi \in^\ell L$. We define $D^s_L := \{\phi \in D^s \mid \phi \in^\ell L\}$.
- D^κ is a nonempty domain of *communities*.
- $U \subseteq W \times D^\kappa \times D^\ell$ is a relation that tracks which languages are *used* by which communities in which worlds.
- $\mathcal{F} : U \to \mathcal{P}(\mathcal{P}(W))$ assigns a nonempty set of *use-facts* $\mathcal{F}(w, c, L)$ to each triple $\langle w, c, L \rangle \in U$, satisfying Facticity, Adequacy, and Closure.
- $C : U \to \mathcal{P}(D^\pi)$ assigns a set of *content assignments* $C(w, c, L)$ to each triple $\langle w, c, L \rangle \in U$, satisfying Supervenience.

To get a feel for how use-models are to be employed, we can consider their application under one particular metaphysical picture of how interpretative constraints are determined, which is presented by Lewis (1975; 1986). The goal here is not to endorse Lewis's metaphysics, but to show that use-models can be employed under familiar metaphysical assumptions.

Let W_{DL} be the set of Lewis-worlds and D^{e}_{DL} the domain of all possibilia—i.e., concrete parts of Lewis-worlds, including individuals, utterances located in space-time, and the worlds themselves.

Here, our "worlds" are Lewis-worlds, so we let $W := W_{\mathrm{DL}}$ and $D^t := \mathcal{P}(W)$. Now, each sentence ϕ is identified with the set $S_\phi \subseteq D^e_{\mathrm{DL}}$ of its token instantiations, and each language L is a set of such sentences. This yields D^s and D^ℓ in the use-model, with $\in^\ell$ interpreted as standard set-membership. Communities pose somewhat of a challenge. Since Lewisian communities (e.g., groups of "flesh-and-blood" speakers) are world-bound, they cannot be directly identified across worlds. So we let each $c \in D^\kappa$ be the set of counterparts of some world-bound community. We can then define U as the set of triples $\langle w, c, L\rangle \in W \times D^\kappa \times D^\ell$ such that c uses L in the Lewis-world w. Conventions of use in Lewis's picture arise from repeated patterns of speaker and hearer behavior, stabilized by common knowledge and governed by norms of "truthfulness and trust" (Lewis, 1975). We can then define $\mathcal{F}(w, c, L)$ as the set of all Lewis-propositions (sets of Lewis-worlds) describing the c's conventions on how to use L in w. These conventions also determine a cluster $C(w, c, L)$ of compatible content assignments (Lewis, 1975, p. 34), each a function from D^s to D^t. Thus, we can define a use-model $\mathcal{M} = \left\langle W, D^\ell, D^s, \in^\ell, D^\kappa, U, \mathcal{F}, C\right\rangle$ within Lewisian metaphysics.

This concludes the reconstruction of a use-model given Lewis's picture of interpretation. However, the filtering lens framework is not restricted to Lewis's metaphysics, and it can plausibly be adapted to accommodate alternative metaphysical approaches to possibility and interpretation (e.g., Stalnaker 2007; 2023). The flexibility of the formal framework thus enables it to clarify the relationship between use-facts and interpretative constraints across diverse metaphysical frameworks, given reasonable assumptions.

3.2 Ways of Use and Interpretative Constraints

Given a use-model $\mathcal{M}$ defined as above, and some triple $\langle w, c, L\rangle$ in U, we can define *c's way of using L in w* as the intersection of all the use-facts about c's use of L in w:

$$u(w, c, L) := \bigcap \mathcal{F}(w, c, L).$$

One should think of $u(w, c, L)$ as the complete story of how c uses L in w, which entails every fact $P \in \mathcal{F}(w, c, L)$ about c's use of L in w, as shown in Figure 1.

To capture how this way of use determines the content of sentences of L, we can define the *interpretative constraint* induced by a given use-fact $P \in \mathcal{F}(w, c, L)$ as:

$$C(c, L|P) := \bigcup_{w' \in P} C(w', c, L).$$

Intuitively, $C(c, L|P)$ is the set of content assignments compatible with c's understanding of L, given that c uses L in a way described by P. Since Closure ensures that $u(w, c, L)$ is itself a use-fact, it will induce some interpretative constraint $C(c, L|u(w, c, L))$, which will be identical to

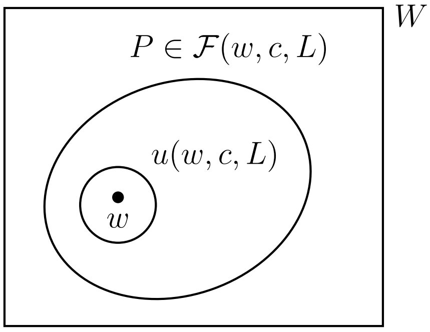

Figure 1: c's way of using L in w, $u(w, c, L)$, and a use-fact $P \in \mathcal{F}(w, c, L)$.

$C(w, c, L)$, due to Facticity and Supervenience. We will identify this set with *c's interpretation of L in w*.

This captures the core claim of the filtering lens picture: that c's interpretation of L in w is the set of all content assignments consistent with c's use of L in w. The interpretative process, on this view, is not a matter of pairing contents with sentences on a one-to-one fashion. Rather, it is a matter of ruling out entire assignments that are incompatible with actual use. The remaining mappings—those in $C(w, c, L)$—are what survive the filter. This picture also provides an alternative way to make sense of the *meaning* (or *content*) of a symbol: while content-based approaches to meaning often identify the meaning of a symbol ε in a language L with what a certain mapping assigns to it, the filtering lens picture suggests a richer notion. Namely, ε's meaning can be identified with (i) a certain use-fact $P_\varepsilon \in \mathcal{F}(w, c, L)$ (or a set of such use-facts) specifying how c uses ε in w, and (ii) the interpretative constraint(s) $C(c, L|P_\varepsilon)$ that are thereby generated.

Notice that this approach naturally allows for multiple viable content assignments, especially when a community's usage does not uniquely determine content. While one might strengthen the filtering lens picture by requiring $C(w, c, L)$ to contain exactly one content assignment for every $\langle w, c, L\rangle \in U$, such uniqueness is not necessary. Allowing multiple content assignments is not a drawback, but a distinctive feature of this framework. Indeed, it provides room to accommodate vagueness, indeterminacy, and other kinds of semantic multiplicity which one often finds in the literature (Van Fraassen, 1966; Fine, 1975; Lewis, 1982).

3.3 On the relata of equivalence judgements

As we have seen, the filtering lens picture provides a new way to understand in virtue of what two theories are equivalent: not by their formal structure, or (primarily) by the fact that they are mapped to the same contents, but because of *how they are used*. An alternative way to understand this shift is to consider what is the *relata* of the equivalence relation under this new picture compared to its predecessors.

Under formal approaches to equivalence, the relata of the equivalence relation are *mathematical structures*, which could be sets of sentences in (uninterpreted) languages, sets of models, or categories. Naturally, formal approaches consider equivalence to be a relation between such structures, which depends on their formal or mathematical properties. Under content-based approaches, on the other hand, the relata of the equivalence relation are sets of sentences in interpreted languages, which can be modeled as *mappings* from sentences to contents. Naturally, content-based approaches consider equivalence to be a relation which depends on the contents that are assigned to the language's sentences (and sub-sentential expressions) by each mapping.

Against these two options, the filtering lens picture suggests that interpreted theories are to be seen as representational systems used by communities in a certain way. An *interpreted language* is thus a triple $\langle w, c, L \rangle$, where w is a world, c is a community, and L is a language used by c in w, and an *interpreted theory* will be a set of sentences $T \subseteq D^s_L$ in an interpreted language $\langle w, c, L \rangle$, being represented by a tuple $\langle w, c, L, T \rangle$. Equivalence between two interpreted theories, in this new picture, will thus depend on how the uses of the two theories are related to each other.

This richer representation of interpreted theories allows us to assess the framework's philosophical substance. In what follows, I apply it to questions of theoretical equivalence, expansion, revision, and quantification.

4 Applications: Theory Structure and Equivalence

I now turn to applications of the filtering lens picture to issues of theory structure and comparison. I begin with a case study: how the framework refines formal criteria of theoretical equivalence, such as definitional and Morita equivalence, by incorporating constraints imposed by use.

4.1 Morita and Definitional Equivalence

Definitional and Morita equivalence are standard formal criteria for theoretical equivalence, particularly within first-order and many-sorted frameworks (Barrett and Halvorson, 2016). These approaches assess equivalence by the existence of a shared definitional extension. But from the perspective of the filtering lens picture, such purely formal criteria are incomplete: they omit the role of use in fixing theoretical content. This section articulates the additional conditions that must be met for a common extension to support a genuine judgment of equivalence.

Before articulating this condition, I should first provide a very brief sketch of definitional and Morita equivalence. This is in no way a detailed account of these notions, but only an outline intended to fix concepts and set the stage for the proposal. The reader is encouraged to consult Barrett and

Halvorson (2016) for a comprehensive account. Both notions work in the context of many-sorted first-order languages, and a theory T in a language L will be a set of sentences of L. I will use $\langle T, L\rangle$ to refer to the theory T in the language L.

Briefly, a many-sorted first-order language is a language L built from symbols belonging to different *sorts*. We start with a set Σ of sort symbols called the *signature* of L, and assign each individual constant c of L a sort σ in Σ, each n-place predicate p of L an *arity* $\sigma_1 \times \cdots \times \sigma_n$, specifying the sorts of the arguments p needs to be combined with to form an atomic sentence, and each n-place function symbol f of L an arity $\sigma_1 \times \cdots \times \sigma_n \to \sigma$, specifying that f is to be combined with n terms of sorts σ_1, …, σ_n to form a term of sort σ. Beyond this, we have the usual logical operators of propositional logic ('$\wedge$', '$\neg$', etc.), the identity predicate '$=$', and "sorted" quantifiers $\ulcorner\forall_\sigma\urcorner$ and $\ulcorner\exists_\sigma\urcorner$ for each sort symbol $\sigma \in \Sigma$. Sentences are formed following the usual rules of first-order logic, but with added restrictions ensuring function and predicate symbols must be applied to terms of the right sort.

These languages can be extended by adding new vocabulary. In this context an *extension* of a theory $\langle T, L\rangle$—be it a definitional or Morita extension—is a theory $\langle T^+, L^+\rangle$ such that L^+'s signature extends L's, $T \subseteq T^+$, and the sentences in $T^+ \setminus T$ are *definitions* of the new vocabulary in L^+ in terms of the old vocabulary in L. Beyond this point, the notions diverge on which new vocabulary is allowed—with only Morita equivalence allowing the addition of new sorts—, but the differences are not relevant for our purposes. Two theories $\langle T_1, L_1\rangle$ and $\langle T_2, L_2\rangle$ are definitionally or Morita equivalent iff they have a common definitional extension, or a common iterated Morita extension $\langle T^+, L^+\rangle$, respectively. This is because, in Barrett and Halvorson's terms, definitional or Morita extensions "say no more" than the original theories they expand (pp. 560–568), and arguably, they also "say no less". Thus, the existence of a common definitional or Morita extension $\langle T^+, L^+\rangle$ for $\langle T_1, L_1\rangle$ and $\langle T_2, L_2\rangle$ will be taken to *say no more and no less* than the original theories themselves, showing that they are equivalent.

But this reasoning overlooks a crucial factor: that equivalence is predicated of interpreted theories. Even if $\langle T_1, L_1\rangle$ and $\langle T_2, L_2\rangle$ are formulated in formally compatible languages, the relevant question is whether the community of users of L_1 and L_2 can use the extension $\langle T^+, L^+\rangle$ in a way that simultaneously (i) *preserves* the use of L_1 and L_2, and (ii) treats the new sentences in T^+ as *definitions* of the new vocabulary in L^+ in terms of the old vocabulary in L_1 and L_2. These constraints may not be jointly satisfiable—a definition that links new terms in L^+ to old ones in L_1 may introduce shifts in use that disrupt inferential or utterance-governing norms in L_2, or vice-versa, even if this disruption is not reflected in the *formal* structure of either language or theory. Thus, the existence of a formal extension is not sufficient for equivalence unless its *use* is appropriately continuous with that of the original theories.

The current framework lets us formally articulate these additional constraints. First, we reconceive a theory not as a pair $\langle T, L\rangle$ of axioms and language, but as a quadruple $\langle w, c, L, T\rangle$, where c is a community using L in world w—i.e. $\langle w, c, L\rangle \in U$—, and T is a set of sentences of L. This shift reflects the main shift in perspective that underlies the filtering lens picture: that interpreted theories are *embodied* or *possibly embodied* in use. Given two theories $\langle w, c, L_1, T_1\rangle$ and $\langle w, c, L_2, T_2\rangle$, the question becomes: is there a world w^+ in which (i) c uses the extension $\langle T^+, L^+\rangle$ in a way that (ii) preserves the prior use of L_1 and L_2, and (iii) treats the added sentences as definitions (or definition-like sentences)? Formally, this means there must be a world w^+ such that:

(i) $\langle w^+, c, L^+\rangle \in U$.

(ii) $w^+ \in u(w, c, L_1) \cap u(w, c, L_2)$.

(iii) For all $i \in C(w^+, c, L^+)$ and all $\phi \in (T^+ \setminus T_1) \cup (T^+ \setminus T_2)$, $i(\phi) = W$.

Given that neither (ii) nor (iii) are guaranteed by the mere *formal* existence of a common definitional or Morita extension, the existence of such extension is not enough to justify a judgment of equivalence. Thus, the filtering lens picture points to conditions like (ii) and (iii) as additions to formal criteria like definitional or Morita equivalence.

Let's take a step back to appreciate the significance of this result. The original motivation for definitional and Morita equivalence was to provide a way to determine the equivalence of two theories by means of a *common extension*. The approach in Barrett and Halvorson (2016) is purely formal, but the paper is riddled with vocabulary that suggests that the equivalence of theories is a matter of *content*. The authors suggest, for instance, that a definitional or Morita extension "says no more" than the original theory it extends. But these remarks are not justified by the formal criteria they provide if taken at face value, since they are about the *content* of the theories, or about their *use*. The filtering lens picture provides a way to make sense of these remarks, by showing that the equivalence of theories is a matter of whether a *common use* of the theories can be found, and that this is not guaranteed by the mere *formal* existence of a common extension. This is a significant result, since it shows that the current picture can be used to enrich formal criteria of equivalence, and that it can provide a way to make sense of the content-based intuitions that are often appealed to in the context of formal criteria of equivalence.

4.2 Theoretical Expansion

Building on the preceding discussion, we can generalize the idea of definitional and Morita extensions on *interpreted theories* to a broader framework. The key idea is to evaluate whether distinct theories can be extended in compatible ways—whether they support *shared trajectories of development* under actual or hypothetical use. This allows us to refine judgments of equivalence, even in cases where standard formal criteria do not apply—e.g. when the theories are not formulated in many-sorted first-order languages.

Formally, given a language L used by a community c in world w, define the set of possible expansions of L—i.e., languages L^+ that extend L in a use-compatible way—as:

$$\mathsf{Cone}(w, c, L) := \{\langle w^+, c, L^+\rangle \in U \mid w^+ \in u(w, c, L) \text{ and } D^s_L \subseteq D^s_{L^+}\}.$$

This set can be visualized as an *inverted cone* on U (Figure 2), with the original use of L at the vertex, and expanding upward toward languages that remain interpretatively compatible with it.[7] Using this concept, the existence of a *common expansion* of two languages L_1 and L_2 corresponds precisely to a common point $\langle w^+, c, L^+\rangle$ in the intersection of their respective cones, $\mathsf{Cone}(w, c, L_1)$ and $\mathsf{Cone}(w, c, L_2)$. This clarifies the conditions added to definitional and Morita equivalence, like condition (ii) in §4.1.

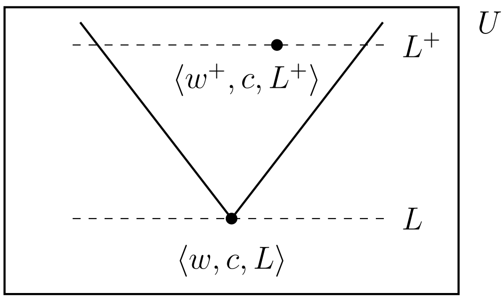

Figure 2: Interpretative expansion space $\mathsf{Cone}(w, c, L)$.

This concept of expansion thus provides a more fine-grained basis for comparing theories than purely formal or coarse content-based methods. Given two theories $\langle w, c, L_1, T_1\rangle$ and $\langle w, c, L_2, T_2\rangle$, we can compare their cones of expansion, $\mathsf{Cone}(w, c, L_1)$ and $\mathsf{Cone}(w, c, L_2)$, which reflect the community's possibilities for extending each theory while preserving its use. These cones are shaped in part by syntax—e.g. use-facts like ($\mathrm{U}_{\wedge\mathrm{E}}$) require that the language contain a two-place operator '$\wedge$'—, but also, crucially, by actual use. Syntax alone does not determine how vocabulary is deployed, and so expansion potential is itself a use-sensitive notion.

This way, the filtering lens framework enhances formal extension criteria like Morita equivalence, enabling us to consider expansions as *actually* or *possibly* embodied in the use of the theories, while accommodating a much broader class of theories than just those defined in many-sorted first-order logic.

[7] I discuss the structure of expansion cones and their interpretation in more detail in other work.

4.3 Theory Revision and Ontic Structural Realism

Ontic Structural Realism (OSR) is the view that the fundamental ontology of the world is structural: fundamentally, *structure is all there is*. The view holds that scientific theories—especially in physics—describe the world by representing its structure, rather than by postulating objects and predicating properties on them (Ladyman, 1998; French and Ladyman, 2003). A central motivation for OSR is the *theory-change argument*: throughout the history of science, major shifts in theoretical ontology (e.g., from Newtonian mechanics to special and then general relativity) have often preserved significant structural features. OSR interprets this structural continuity as tracking a real feature of the world—namely, its objective structure. Since structure, as opposed to "ontology", is often the only aspect that survives radical theory change, OSR concludes that it is the only component deserving metaphysical commitment. This argument has been taken up by contemporary advocates such as Wallace (2022), who frames OSR as a part of, or a result of, a "math-first" approach to physical theories. On Wallace's formulation, the structural features of a theory are captured in its *mathematical formulation*, which sits well with formalist approaches to the question of content.

But there is an alternative explanation as to why the structure of theories is retained in theory change. Theories are *tools*, and when we adopt a new theory or change an existing one, we want to be able to do many of the things that we did with the old theory. Because of this, the old and the new theory need to share some structure: just as a new hammer needs to be at least somewhat similar to the old one to serve the same purposes, a new theory needs to resemble the old one in many ways in order to fulfill many of the same purposes when it comes to prediction, inference, and explanation. This alternative explanation is based on the idea that theories are *used* by communities, and that what is really retained through theory change is (part of) this use. To the extent that this use requires the new theory to retain some of the structure of the old theory, there will be pressure to retain that structure under theory change. Notice that this explanation is *compatible* with OSR, but doesn't *require* it. To the extent that the proponent of OSR can connect this explanation with the idea that theories "latch onto" the structure of the world (and only to it), they can use it alongside OSR; but critics of OSR can *also* appeal to it and argue that the specific theory-world connection postulated by OSR is unnecessary for explaining the retention of structure under theory change.

Regardless of who ends up using it, the filtering lens framework enables us to provide a *formal account of theory revision*, something much needed in these debates. The key idea is that revision is anchored in the partial retention of prior use. The more use-facts retained, the closer the revised theory remains to the original. Recall from previous sections that a theory can be represented as a tuple $\langle w, c, L, T \rangle$, where c is a community using language L in world w, and T is a set of sentences of L. The use-facts $\mathcal{F}(w, c, L)$ capture the interpretative practices governing this theory. To model revision, we consider *weakening* the theory by deleting some of these use-facts. Formally, a weakening is any proper subset $\mathcal{G} \subsetneq \mathcal{F}(w, c, L)$. Intuitively, $\mathcal{G}$ represents a less constrained way of using L—a looser conventional basis. This allows more mappings from sentences to contents, i.e., more content assignments. It also enables reinterpretation and expansion paths not available under the original constraints. A *revision of the theory based on* $\mathcal{G}$ is then a new theory $\langle w', c, L', T' \rangle$, where $w' \in \bigcap \mathcal{G}$. We might additionally require some syntactic or structural continuity between $\langle L, T \rangle$ and $\langle L', T' \rangle$, but the kind of continuity that is required highly depends on the specific use-case.

This gives us a natural ordering of revisions by proximity. Given two weakenings $\mathcal{G}_1 \subseteq \mathcal{G}_2$, they determine the regions $\bigcap \mathcal{G}_1$ and $\bigcap \mathcal{G}_2$ around the original theory. The larger region of the weakening, the larger the set of worlds compatible with the revision, and the further we move from the original interpretation (Figure 3). Thus, we have a principled way to measure semantic distance between theories which does not consist in the all-or-nothing preservation of the original ontology, but which also acknowledges that more than structure needs to be preserved. While syntactic and mathematical structure may affect what expansions or reinterpretations are viable, interpretative distance is mainly a function of *use*. This allows the framework to recover the idea that successor theories preserve *something* of their predecessors without committing to structure as the only ontologically privileged component.

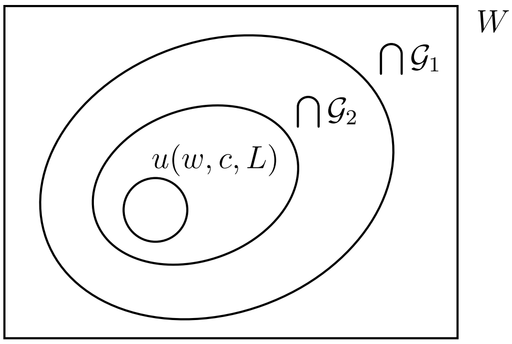

Figure 3: Regions determined by weakenings $\mathcal{G}_1$ and $\mathcal{G}_2$ of a theory $\mathbf{T} = \langle w, c, L, T \rangle$.

5 Applications: Semantics and Metaphysics

The filtering lens picture can also be used to address problems in semantics and metaphysics. In this section, I will show how it can further justify alternative accounts of quantification like Rayo's domain-free semantics (2017), and how it can clarify metametaphysical disputes like those in Hirsch (2002) and Sider (2009).

5.1 Domain-Free Semantics

To motivate the next application, I must first explain *domain-free semantics*—a semantic framework intended to model quantificational meaning without relying on a fixed domain of entities. This framework was introduced by Rayo (2017) to argue against *absolutism*, the view that there is a sense to be made of discourse concerning "absolutely everything". By constructing such a framework, he intends to develop a "facts-first", instead of "objects-first", conception of how language relates to the world. This allows him to argue that absolutism, and the reliance on objectual domains more generally, is not imposed on us, but is rather a result of a certain way of doing semantics.

The conception Rayo develops resembles the standard one in that sentences represent facts or express propositions, and so it involves a mapping from sentences to propositions. The difference is that, in the standard conception, this mapping has to be done by first specifying a domain of objects, and then mapping the individual constants to these objects. According to Rayo, this way of assigning sentential content is just a middleman: it is a means for assigning propositions to sentences, which is the real goal of semantics (Rayo, 2017, p. 255). I will not take a stand on whether Rayo's argument against the absolutist is successful, but it is useful to note that the formal tool he relies on, domain-free semantics, can be recovered from the structure of use-models. This allows us to further justify Rayo's approach, and relate it to our representational practices. I will sketch how this can be done, and explain why it is useful, in the following paragraphs.

Rayo (2017) considers the case of a first-order language L with individual constants $a_0, a_1, \ldots$ and predicates $P, Q, R, \ldots$—including the identity predicate '$=$'—, and the usual logical vocabulary of connectives, variables, and quantifiers. On the standard conception, this assignment is fixed by a *Kripke structure*: a tuple $\mathcal{K} = \langle W, a, D, \mathfrak{d}, \mathfrak{c}, \mathfrak{p} \rangle$, where W is a set of worlds, $a \in W$ is the "actual" world, D is a domain of objects, $\mathfrak{d}$ is a function that assigns a set $D_w \subseteq D$ to each world $w \in W$, $\mathfrak{c}$ is a (possibly partial) function that assigns elements of D to the individual constants of L, and $\mathfrak{p}$ is a function that assigns a set of n-tuples of D_w to each pair $\langle w, \Pi \rangle$ of a world $w \in W$ and an n-place predicate Π of L. Given $\mathcal{K}$, we can construct an assignment $i_{\mathcal{K}}$ of propositions to formulas ϕ of L, roughly as follows:

- A *sequence* σ of L is a mapping from the set of variables of L to D. Given such a sequence, we define the *denotation function* δ_σ as a mapping from the set of variables and constants of L to D by $\delta_\sigma(\alpha) := \sigma(\alpha)$ if α is a variable, or $\mathfrak{c}(\alpha)$ if α is a constant.
- If $\phi = \ulcorner \Pi(\alpha_1, \ldots, \alpha_n) \urcorner$ is an atomic sentence where Π is an n-place predicate and $\alpha_1, \ldots, \alpha_n$ are variables or constants, then we define

$$i^{\sigma}_{\mathcal{K}}(\phi) := \{w \in W \mid \langle \delta_\sigma(\alpha_1), \ldots, \delta_\sigma(\alpha_n) \rangle \in \mathfrak{p}(w, \Pi)\}.$$

- $i_{\mathcal{K}}^{\sigma}(\ulcorner\neg\phi\urcorner) := W \setminus i_{\mathcal{K}}^{\sigma}(\phi)$, $i_{\mathcal{K}}^{\sigma}(\ulcorner\phi\vee\psi\urcorner) := i_{\mathcal{K}}^{\sigma}(\phi) \cup i_{\mathcal{K}}^{\sigma}(\psi)$, and so on for the other connectives.
- If v is a variable, $i_{\mathcal{K}}^{\sigma}(\ulcorner\exists v\phi\urcorner) := \{w \in W \mid w \in i_{\mathcal{K}}^{\sigma'}(\phi)$ for some sequence σ' that agrees with σ on all variables distinct from $v\}$.

This suffices to define an assignment $i_{\mathcal{K}}$ of propositions to sentences of L—formulas without free variables: we define $i_{\mathcal{K}}(\phi) := p$ iff $p = i_{\mathcal{K}}^{\sigma}(\phi)$ for any sequence σ of L. Note, also, that the "meaning" of a predicate P can be identified with the set of n-tuples $\{\langle w, \mathfrak{p}(w, P)\rangle \mid w \in W\}$, and the "meaning" of a constant a_n can be identified with the element $\mathfrak{c}(a_n) \in D$.

Rayo eschews the use of Kripke structures in favor of a more direct approach to assigning propositions to sentences—one that does not require the use of a domain of objects D. Instead, he uses structures of the form $\mathcal{R} = \langle I, \iota, \circ\rangle$, where I is a set of *atomic assignments* of L—functions that assign subsets of W to atomic sentences of L—, $\iota \in I$ is a distinguished member of I, and $\circ : I \times I \to I$ is a function that "merges" two atomic assignments. This structure is required to satisfy some conditions whose formal specification would take us too far afield, but which roughly ensure that:

(i) Every $\mathfrak{a} \in I$ respects the logical properties of '$=$'.

(ii) For any $\mathfrak{a}, \mathfrak{b} \in I$, $\mathfrak{a} \circ \mathfrak{b}$ "combines" $\mathfrak{a}$'s and $\mathfrak{b}$'s interpretations of the individual constants, by interpreting the constant a_{2n} the way $\mathfrak{a}$ interprets constant a_n and interpreting a_{2n+1} the way $\mathfrak{b}$ interprets constant a_n.[8]

Intuitively, the assignments in I are those that respect our understanding of the *predicates* of L, and the assignment $\iota \in I$ is the one in I that *additionally* respects our understanding of the *individual constants* of L (p. 255). Given $\mathcal{R} = \langle I, \iota, \circ\rangle$, one can define a mapping $i_{\mathcal{R}}$ of propositions to sentences of L. What we need to do, first, is to define an assignment $i_{\mathcal{R}}^{\mathfrak{a}}$ of propositions to sentences of L *relative to* an atomic assignment $\mathfrak{a}$. This is done roughly as follows, where $\mathfrak{a}$ and $\mathfrak{b}$ are any atomic assignments:

- If ϕ is an atomic sentence, then $i_{\mathcal{R}}^{\mathfrak{a}}(\phi) := \mathfrak{a}(\phi)$.
- $i_{\mathcal{R}}^{\mathfrak{a}}(\ulcorner\neg\phi\urcorner) := W \setminus i_{\mathcal{R}}^{\mathfrak{a}}(\phi)$, $i_{\mathcal{R}}^{\mathfrak{a}}(\ulcorner\phi \vee \psi\urcorner) := i_{\mathcal{R}}^{\mathfrak{a}}(\phi) \cup i_{\mathcal{R}}^{\mathfrak{a}}(\psi)$, and so on for the other connectives.
- If v is a variable, $i_{\mathcal{R}}^{\mathfrak{a}}(\ulcorner\exists v\phi\urcorner) := \bigcup\{i_{\mathcal{R}}^{\mathfrak{b}}(\ulcorner a_k = a_k\urcorner) \cap i_{\mathcal{R}}^{\mathfrak{b}}(\ulcorner\phi[v/a_k]\urcorner) \mid \mathfrak{b} \in I$ agrees with $\mathfrak{a}$ on all atomic sentences not involving $a_k\}$, for some (any) individual constant a_k that does not occur in ϕ, where $\phi[v/a_k]$ is the formula ϕ with free occurrences of v replaced by a_k.

[8] This condition supervenes on the way the atomic *sentences* are interpreted by $\mathfrak{a}$, $\mathfrak{b}$, and $\mathfrak{a} \circ \mathfrak{b}$, so no assignment of semantic values to the *constants* is required to articulate it. See the appendix in Rayo (2017) for the formal specification of this condition.

Explicitly formalizing the last condition requires the use of $\circ$, but it can stay like this for present purposes. Rayo shows that any "interpretation" of L—i.e. any mapping from its sentences to propositions—which can be obtained using regular domain-based semantics (Kripke structures) can also be obtained using domain-free semantics, and vice-versa. This shows that domain-based semantics is not necessary, as long as the goal is to assign propositions to sentences of L. The mapping $i_{\mathcal{R}}$ is then specified by defining $i_{\mathcal{R}}(\phi) := i^{\iota}_{\mathcal{R}}(\phi)$ for any sentence ϕ of L.

But one can still wonder how we connect domain-free semantics for L with our representational practices when using L. Here, domain-based semantics seems to have an advantage: an assignment of propositions to sentences that starts with a domain of *entities* and then assigns *properties* to these entities seems to better capture our *practice* of using L to represent the world: we refer to objects and ascribe properties to them using the constants and predicates of L. Even more, it seems that domain-based semantics has the ability to capture the *meaning* of the predicates and individual constants of L via its components $\mathfrak{p}$ and $\mathfrak{c}$, as we have seen above, while domain-free semantics does not have any component that can be naturally associated with these meanings.

Fortunately, the filtering lens picture can help us make sense of domain-free semantics in an analogous way by connecting it with our *use* of the language. This is done by obtaining the structure $\mathcal{R} = \langle I, \iota, \circ \rangle$ out of a tuple $\langle w, c, L \rangle \in U$ in an appropriate use-model. Formally, in an interpretation $\langle I, \iota, \circ \rangle$ for L, I could be *any* set of atomic assignments, ι any member of I and $\circ$ any dyadic function on I, as long as the formalizations of (i) and (ii) hold. But recall that a structure $\mathcal{R} = \langle I, \iota, \circ \rangle$ is said to represent *our* understanding of L if I is the set of atomic assignments that respect our understanding of the *predicates* of L, and ι the atomic assignment in I that additionally respects our understanding of the *individual constants* of L (Rayo, 2017, pp. 255). This restriction is only put forth as an intuitive gloss in Rayo's paper, and not as a formal requirement, but it can be made precise by the filtering lens picture. Additionally, the filtering lens picture allows us to make sense of the meaning of constants and predicates in L in domain-free semantics.

In this framework, we can represent a community c's understanding of L as consisting in certain *use-facts* $P, K \in \mathcal{F}(w, c, L)$, where P describes how c uses the predicates, and K describes how c uses the individual constants of L. Given that extensions of atomic assignments in I respect the properties of the logical operators of L—i.e. of '$=$', '$\neg$', '$\vee$', '$\exists$', etc.—we need to posit an additional use-fact $T \in \mathcal{F}(w, c, L)$ that describes how c uses this logical vocabulary. The set I of atomic assignments in domain-free semantics can then be defined as follows:

$$I := \{\mathrm{At}_L(i) \mid i \in C(c, L | P \cap T)\},$$

where $\mathrm{At}_L(i)$ is the restriction of i to atomic sentences of L. As per §3.2, this set is obtained from the interpretative constraint given by the use-facts P and T, which capture c's understanding of the predicates and the logical vocabulary of L. The atomic assignment ι is then taken to satisfy the following condition:

$$\iota \in \{\mathrm{At}_L(i) \mid i \in C(c, L|P \cap K \cap T)\} \subseteq I,$$

which means that ι is the atomic assignment in I that *additionally* respects c's understanding of the individual constants of L. When developing domain-free semantics, Rayo assumes that ι can be singled out by this condition, which would make the set $\{\mathrm{At}_L(i) \mid i \in C(c, L|P \cap K \cap T)\}$ a singleton. But this is not a requirement of the current framework, and in some cases (e.g. vagueness or semantic indeterminacy) it may not hold.

Given I and ι, we can define $\circ$ using I and the syntax of L as it is done in Rayo (2017, pp. 267–268). This allows us to get the interpretation $\langle I, \iota, \circ\rangle$ from $\langle w, c, L\rangle$ and the use-facts $P, K, T \in \mathcal{F}(w, c, L)$, and thereby to connect domain-free semantics with c's representational practices. In addition to this, we have a way to define structures which can naturally be taken to correspond to the meanings of predicates and constants in L. Recall from §3.2 that we represent the meaning of expressions via their associated use-facts and constraints. Given this, we can consider the use-fact P as being decomposable into use-facts P_Π for individual predicates Π of L, and the use-fact K as being decomposable into use-facts K_c for individual constants c of L. This allows us to account for both the intuitive gloss of domain-based semantics given above, and the question of how we can make sense of the meaning of constants and predicates of L in domain-free semantics, eliminating the perceived advantage domain-based semantics has in this regard.

Of course, we can't use just *any* use-model for this purpose. For instance, if $\langle w, c, L\rangle$ is such that I has "too few" assignments, we may not be able to define $\circ$ in a way that satisfies (i) and (ii). On the other hand, if I has "too many" assignments, we may not be able to single out ι. But these constraints can naturally be seen as the demand that $\langle w, c, L\rangle$ must adequately represent c's use of L. Thus, to the extent that this demand is met, the current framework provides a way to connect domain-free semantics with a community's representational practices, and thereby to make sense of Rayo's approach in a way that is sensitive to the way language is used.

5.2 Second-Order Quantification

We have seen how the filtering lens picture allows us to connect domain-free semantics with a given community's representational practices. It turns out that this framework can also be used to *extend* the domain-free account of quantification to second-order quantification, and possibly to other kinds of higher-order quantification.

Recall that to make sense of Rayo's account, we needed the use-facts $P, K, T \in \mathcal{F}(w, c, L)$, where P describes how c uses the predicates of L, K describes how c uses the individual constants of L, and T describes how c uses the logical vocabulary of L. Given this, we can generalize the account of first-order quantification by considering a set I^2 of atomic assignments which respect the use of individual constants and the logical vocabulary, but vary the use of the predicates:

$$I^2 := \{\mathrm{At}_L(i) \mid i \in C(c, L|K \cap T)\},$$

and obtain the distinguished assignment ι by adding the use-fact P, which describes how c uses the predicates of L:

$$\iota \in \{\mathrm{At}_L(i) \mid i \in C(c, L|P \cap K \cap T)\} \subseteq I^2.$$

Again, this means that ι is the atomic assignment in I^2 that *additionally* respects c's understanding of the predicates of L. As before, we can either follow Rayo and posit that ι can be singled out by this condition, or we can allow for the possibility that it is not a singleton.

We can then define the interpretation of second-order-quantified sentences analogously to how we defined the interpretation of first-order-quantified sentences in §5.1, where X^n is an n-place *predicative variable*, by the condition:

- $i^{\mathfrak{a}}(\ulcorner \exists X^n \phi(X^n) \urcorner) = \bigcup \{ i^{\mathfrak{b}}(\ulcorner \phi(P^n) \urcorner) \mid \mathfrak{b} \in I^2$ agrees with $\mathfrak{a}$ on atomic sentences not involving $P^n\}$, for some (any) n-adic predicate P^n that does not occur in $\phi(X^n)$.[9]

This approach unifies first- and second-order quantification under a common framework: both are modeled by structures obtained by fixing the use of certain syntactic expressions and letting the use of other syntactic expressions vary. Additionally, the naturalness of the move from first- to second-order quantification hints at a more general account which includes other kinds of higher-order quantification (Bacon, 2023), although I will not pursue this here.

5.3 Quantifier Variance and Collapse

The above treatment of quantification supports a picture according to which it isn't fixed by a "metaphysically distinguished" domain of objects, but by a range of content-assignments compatible with a given set of use-facts. A natural question is, then, whether the current framework can be used to defend another "deflationary" view of quantification: *quantifier variance*. This is the view that there is no single, metaphysically privileged meaning of quantificational expressions like 'there is' or '$\exists$'. Instead, the meaning of these expressions is determined by the way they are used in a community, and different communities may use them in different ways, which are equally good for the purposes of metaphysics (Hirsch 2002; 2005; 2009).

[9] Note that, as above, this condition can be formally expressed using an analogue $\bullet$ of $\circ$ in §5.1, but on predicates instead of constants. The basic idea, assuming that there are countably many predicates $P^n_0, P^n_1, \ldots$ of any arity n in L, is that $\mathfrak{a} \bullet \mathfrak{b}$ interprets P^n_{2k} the way $\mathfrak{a}$ interprets P^n_k, and P^n_{2k+1} the way $\mathfrak{b}$ interprets P^n_k, for any n and k. As footnote 8 explains, this condition supervenes on the way the atomic *sentences* are interpreted by $\mathfrak{a}$, $\mathfrak{b}$, and $\mathfrak{a} \bullet \mathfrak{b}$, so no assignment of semantic values to the *predicates* is required to articulate it.

To illustrate the point, we might consider two hypothetical languages which are phonetically and syntactically identical to English, but in which the expression 'there is' has different meanings. I will follow Sider (2022) in calling these languages *Nihilese* and *Universalese*. Speakers of Nihilese take a sentence to hold just in the situations a(n idealized) English-speaking mereological nihilist would accept it as true. Similarly, speakers of Universalese take a sentence to hold just in the situations a(n idealized) English-speaking mereological universalist would accept it as true. So, for instance, speakers of Universalese would take the sentence 'There is a chair' to hold just in the situations in which some particles are arranged chair-wise, but speakers of Nihilese would take this very sentence to be false in the same situations. Crucially, the variantist claims that the acceptance or rejection of sentences like these by speakers of the two languages is not due to any disagreement about how the world is, but rather to a difference meaning. In particular, it is due to a difference in what the expression 'there is' means in each language.

Translating this example to the current framework, we can consider the syntactical part of a language L resembling English, or a regimented first-order fragment of it. We can suppose that there are two communities c_N and c_U that use L in different ways, both within the same world w, which yields two different triples $\mathcal{L}_N = \langle w, c_N, L \rangle$ and $\mathcal{L}_U = \langle w, c_U, L \rangle$, corresponding to Nihilese and Universalese, respectively, in this simplified setting. The claim that these languages are equally good for doing metaphysics can be understood, within the present framework, as a claim about *sentential content*: roughly, that for any sentence of L that receives a definite content in one interpreted language, there is some sentence (not necessarily the same one) that receives that same content in the other.[10] In this sense, the two languages are able to describe the same facts, or express the same propositions, albeit using different sentences in each case.

The further claim that '$\exists$' functions as an existential quantifier in both languages is naturally understood in terms of inferential role. Thus, the variantist can say that '$\exists$' in a language $\mathcal{L}$ is taken to have the meaning of an existential quantifier whenever the syntactic consequence relation $\vdash$ of $\mathcal{L}$ satisfies the following conditions:

($\exists$I) $\phi \vdash \ulcorner \exists v \phi' \urcorner$, where ϕ is a sentence of $\mathcal{L}$, v any variable of $\mathcal{L}$, and ϕ' is like ϕ but for the replacement of zero or more occurrences of some name a in ϕ with free occurrences of v.

($\exists$E) If $\phi \vdash \psi$, and ϕ' is like ϕ but for the replacement of zero or more occurrences of some name a that does not occur in ψ with free occurrences of v, $\ulcorner \exists v \phi' \urcorner \vdash \psi$.

[10] That a given language $\langle w, c, L \rangle$ assigns a definite content to a sentence ϕ is to be understood along the lines of (Δ). Of course, this is a simplification, as there may be vague or otherwise semantically indeterminate sentences that are nonetheless *meaningful* in $\langle w, c, L \rangle$. But the issue of how to handle such sentences is not relevant to the present discussion.

Translating this into the present framework, we can say that $\mathcal{L} = \langle w, c, L \rangle$ uses '$\exists$' as an existential quantifier, in this minimal sense, just in case every admissible content assignment $i \in C(w, c, L)$ satisfies the following conditions:

($C^{L}_{\exists I}$) $i(\phi) \subseteq i(\ulcorner \exists v \phi' \urcorner)$, where ϕ is a sentence of L, v any variable of L, and ϕ' is like ϕ but for the replacement of zero or more occurrences of some name a in ϕ with free occurrences of v.

($C^{L}_{\exists E}$) If $i(\phi) \subseteq i(\psi)$, and ϕ' is like ϕ but for the replacement of zero or more occurrences of some name a that does not occur in ψ with free occurrences of v, then $i(\ulcorner \exists v \phi' \urcorner) \subseteq i(\psi)$, for any sentences ϕ and ψ of L.

This provides a first pass at what it is for '$\exists$' to count as an existential quantifier, and it suffices to articulate the variantist's initial claim that both $\mathcal{L}_N$ and $\mathcal{L}_U$ employ existential quantification. For the variantist's case to get off the ground, however, she needs to show that despite the fact that both languages satisfy these constraints, they can assign different truth-conditions to some sentences involving '$\exists$', just like the case of 'There is a chair' in Nihilese and Universalese. We will skip this extra step for now and simply assume that there are some existential statements which are given different truth-conditions in each language, and that this is *only* due to a difference in use-facts involving '$\exists$' in each language.

This inferential-role characterization, however, is widely thought to be insufficient. A prominent version of this objection is due to Dorr (2014), who argues that local introduction and elimination rules cannot fully capture the meanings of expressions like '$\exists$'. His argument starts with the observation that competent speakers seem to grasp much more than local introduction and elimination rules like ($\exists$I) and ($\exists$E) encode. To advance his argument, he makes an analogy with disjunction ('$\vee$'), where the rules for disjunction for sentences of a language $\mathcal{L}$ are given by:

($\vee$I) $\phi \vdash \ulcorner \phi \vee \psi \urcorner$ and $\psi \vdash \ulcorner \phi \vee \psi \urcorner$.

($\vee$E) If $\phi \vdash \chi$ and $\psi \vdash \chi$, then $\ulcorner \phi \vee \psi \urcorner \vdash \chi$.

Dorr rightly notes that these rules are not enough to capture the meaning of '$\vee$' in the languages we use. Indeed, whatever the semantic value of '$\vee$' happens to be, we know that for any planet x, *including those we have not named*, it maps the proposition *that x is rocky* and the proposition *that x is inhabited* onto the proposition *that x is rocky or x is inhabited*—a proposition that is entailed by the proposition *that x is rocky* and the proposition *that x is inhabited*, and that entails every proposition that both of them entail (Dorr, 2014, p. 511). But, of course, this is not captured by the local rules for disjunction, since they only operate on the sentences that a language contains. Thus, to account for the full meaning of '$\vee$', Dorr argues that we need to posit a richer semantic framework. In particular, we need to assign '$\vee$' to a two-place function $f_{\vee}$ that maps propositions to propositions, such that $f_\vee(P, Q)$ is entailed by P and Q, and entails every proposition that both P and Q entail, for any propositions P and Q. Thus, if P is the proposition *that* x *is rocky* and Q is the proposition *that* x *is inhabited*, then $f_\vee(P, Q)$ is the proposition *that* x *is rocky or* x *is inhabited*. Since the present framework uses possible world semantics for simplicity, we can think of $f_\vee$ as the function such that $f_\vee(P, Q) = P \cup Q$, for any two sets of worlds P and Q. Naturally, the semantic value a given mapping $[\![\cdot]\!]$ assigns to sentences of the form $\ulcorner \phi \vee \psi \urcorner$ is then given by $f_\vee([\![\phi]\!], [\![\psi]\!])$.

Now, we can extend this argument to quantification. Much for the same reasons, we seem to grasp much more about 'there is' or '$\exists$' than the local rules for quantification like ($\exists$I) and ($\exists$E) capture. Indeed, whatever the semantic value of '$\exists$' happens to be, we seem to know that for any *concept* $\mathfrak{c}$, including those we don't have predicates for, '$\exists$' seems to map $\mathfrak{c}$ to the proposition *that* $\mathfrak{c}$ *is satisfied*.[11] Indeed, if we *had* a predicate 'F' expressing $\mathfrak{c}$, then '$\exists x Fx$' would be evaluated to this proposition. First, notice that this is not captured by the local rules for quantification, since they only operate on the sentences that a language contains. Thus, Dorr argues, we need the richer semantic framework of *concepts* to account for the full meaning of '$\exists$'. In particular, we need to assign '$\exists$' to a function $f_\exists$ that maps concepts to propositions, such that $f_\exists(\mathfrak{c})$ is the proposition *that* $\mathfrak{c}$ *is satisfied*, for any concept $\mathfrak{c}$.[12] Even more, once a space of concepts $\mathfrak{C}$ is fixed, the relevant function $f_\exists$, and thus the semantic value of '$\exists$' is uniquely determined by its behavior on $\mathfrak{C}$.[13]

This leaves the variantist with a problem. Recall that the differences in meaning between the two languages are, by stipulation, only due to a difference in use-facts involving '$\exists$' in each language. Thus, for a given sentence '$\exists x Fx$', 'F' must be understood as expressing the same concept in both languages, say, $\mathfrak{c}$. But then it at least one of the two languages, '$\exists x Fx$' must be evaluated to a (coarse-grained) proposition different than $f_\exists(\mathfrak{c})$, which means that '$\exists$' must be understood in terms *other* than $f_\exists$. Thus, either one of the two linguistic communities is not using '$\exists$' as an existential quantifier, or they are operating on different *spaces of concepts*. In the former case, quantifier variantism is simply false, since it requires the languages in question to use '$\exists$' as an existential quantifier. In the latter case, the argument that the relevant languages are equally good for doing metaphysics is significantly weakened, since there are aspects of the world that are not shared by the two languages—in particular, the two languages are not able to express the same concepts.

The present framework agrees with Dorr on one crucial point: local inferential rules like ($\exists$I) and ($\exists$E) are too weak to capture the meanings of expressions like '$\exists$'. Where it departs from Dorr

[11] Perhaps $\mathfrak{c}$ is having a particular hue of red which we haven't named yet.

[12] This is a simplification for the sake of exposition. In the real account, the semantic value of '$\exists$' maps $(n+1)$-adic concepts to n-adic concepts, but this doesn't make a difference here.

[13] Let $\mathfrak{C}$ be a space of concepts. In Dorr's framework, concepts have an entailment relation between them—call this relation *concept-entailment*. Now, for any proposition P, define Exp(P) as the concept *being such that* P *holds*. Here Exp(P) is satisfied by everything if P holds, and by nothing if P fails to hold. Given all this, we can define a function $f_\exists$ from concepts to propositions such that (i) $\mathfrak{c}$ concept-entails Exp$(f_\exists(\mathfrak{c}))$, and (ii) whenever $\mathfrak{c}$ concept-entails Exp(P), $f_\exists(\mathfrak{c})$ entails P. For more details, see Dorr (2014) and Sider (2022).

is in its diagnosis of what is missing. Rather than introducing a space of concepts, it provides a way of capturing what is missing from local rules like (∃I) and (∃E) by positing that the relevant constraints on content assignments are *stable* under certain kinds of expansion, as understood in §4.2. Thus, facts about how a community uses an expression do not merely constrain admissible content assignments to the sentences currently available in the language; they also constrain how that expression must behave when the language is extended in ways that preserve prior use. In other words, meaning includes not just local inferential constraints, but constraints that hold across expansions of the language.

Schematically, let P_ε be a use-fact governing an expression ε in $\langle w, c, L\rangle$. Then P_ε determines not only constraints on content assignments via $C(c, L|P_\varepsilon)$, but also constraints on content assignments via $C(c, L^+|P_\varepsilon)$ for any admissible expansion $\langle w^+, c, L^+\rangle$ of $\langle w, c, L\rangle$ in the sense of §4.2. Thus, in the case of existential quantification, the fact '∃' is used as an existential quantifier in some language $\langle w, c, L\rangle$ determines not only the constraints ($\mathrm{C}^{L}_{\exists\mathrm{I}}$) and ($\mathrm{C}^{L}_{\exists\mathrm{E}}$), but it thereby also determines the following constraints on any content assignment i compatible with the use of any expansion $\langle w^+, c, L^+\rangle$ of $\langle w, c, L\rangle$:

($\mathrm{C}^{L+}_{\exists\mathrm{I}}$) $i(\phi) \subseteq i(\ulcorner \exists v \phi' \urcorner)$, where ϕ is a sentence of L^+, v any variable of L^+, and ϕ' is like ϕ but for the replacement of zero or more occurrences of some name a in ϕ with free occurrences of v.

($\mathrm{C}^{L+}_{\exists\mathrm{E}}$) If $i(\phi) \subseteq i(\psi)$, and ϕ' is like ϕ but for the replacement of zero or more occurrences of some name a that does not occur in ψ with free occurrences of v, then $i(\ulcorner \exists v \phi' \urcorner) \subseteq i(\psi)$, for any sentences ϕ, ψ, and v of L^+.

We can thereby characterize what is it for '∃' to be used as an existential quantifier as there being a given use-fact $P_\exists \in \mathcal{F}(w, c, L)$ which guarantees that the relevant constraints ($\mathrm{C}^{L+}_{\exists\mathrm{I}}$) and ($\mathrm{C}^{L+}_{\exists\mathrm{E}}$) are satisfied for any expansion. Thus, these constraints should be satisfied by any content assignment $i \in C(c, L^+|P_\exists)$.

This case is exactly analogous to the case of disjunction. Here, for '∨' to be used as a disjunction in some language $\langle w, c, L\rangle$ is for there to be a use-fact $P_\vee$ such that for any extension $\langle w^+, c, L^+\rangle$ of $\langle w, c, L\rangle$ and any assignment i in $C(c, L^+|P_\vee)$, we have that:

($\mathrm{C}^{L+}_{\vee\mathrm{I}}$) $i(\phi) \subseteq i(\ulcorner \phi \vee \psi \urcorner)$ and $i(\psi) \subseteq i(\ulcorner \phi \vee \psi \urcorner)$, where ϕ and ψ are any sentences of L^+.

($\mathrm{C}^{L+}_{\vee\mathrm{E}}$) If $i(\phi) \subseteq i(\chi)$ and $i(\psi) \subseteq i(\chi)$, then $i(\ulcorner \phi \vee \psi \urcorner) \subseteq i(\chi)$, for any sentences ϕ, ψ, and χ of L^+.

Now, consider our initial example: the fact that '∨' is understood to map the proposition *that x is rocky* and the proposition *that x is inhabited* to the proposition *that x is rocky or x is inhabited*. This fact cannot be captured by the local rules for disjunction, but it *can* be captured by the constraints ($C^{L+}_{\vee I}$) and ($C^{L+}_{\vee E}$). To show this, if suffices to note that if $\langle w, c, L\rangle$ is the starting language, there is a possible expansion $\langle w^+, c, L^+\rangle$ of it in which a name n is added, together with the use-fact P_n specifying that n is to be used as a name for the planet x. Dorr's observation is accounted for by the fact that for any content assignment i compatible with $\langle w^+, c, L^+\rangle$, we have that $i(\ulcorner \phi_n \vee \psi_n \urcorner)$ satisfies ($C^{L+}_{\vee I}$) and ($C^{L+}_{\vee E}$).

In both cases, the extra conditions are not captured by local elimination and introduction rules, since they only concern the sentences of the language, and not its possible expansions. But also, in both cases, we are able to capture the information missing from the local rules without introducing an extra realm of semantic values, like propositional functions or concepts. Thus, the current framework undercuts Dorr's argument against quantifier variantism by providing an alternative way of capturing facts about meaning.

The current framework also clarifies a puzzle about quantifier variantism. At first glance, there is a tension between the claims that (i) '$\exists$' is an existential quantifier in both $\mathcal{L}_N$ and $\mathcal{L}_U$, and (ii) '$\exists$' differs in meaning between the two languages. If the meaning of '$\exists$' were exhausted by a single use-fact $P_\exists$, this would indeed suggest that the meanings coincide. The tension dissolves once we accept the possibility of there being multiple use-facts that govern the employment of a given expression in the same language. What is required for '$\exists$' to count as an existential quantifier is that the total use-facts governing its employment entail a core fact $P^{\text{min}}_\exists$ that guarantees the conditions ($C^{L+}_{\exists I}$) and ($C^{L+}_{\exists E}$). Different communities may then impose additional use-facts—concerning, for example, when a given speaker is licensed to use a given expression in a given context—that further constrain admissible interpretations. These additional constraints may differ between $\mathcal{L}_N$ and $\mathcal{L}_U$, yielding genuinely different quantifier meanings, even though both count as *existential quantifiers*.

Needless to say, this does not preclude the possibility of quantifier variantism being wrong for independent reasons. Perhaps there are independent ways of establishing that '$\exists$' cannot really differ in meaning without ceasing to count as an existential quantifier, or perhaps there are independent reasons to think that the two languages cannot really express the same set of (coarse-grained) *sentential* contents, or are not equally good for doing metaphysics for other reasons. Nevertheless, I take it to at least have shown a way forward for the quantifier variantist.

6 Conclusion

The ways in which a community employs a language bridge the apparent gap between content and syntax, which is missing in both purely formal and purely content-based approaches to interpretation and equivalence. Once use is brought to the foreground, it is clear how formal features of a theory may help determine its content—but only insofar as they are embedded in the representational practices of a community. Conversely, we avoid problems of overgeneration or direction of explanation when interpretative space is seen as constrained by facts about use. By recognizing that syntax and content are connected through patterns of use, we recover what is most compelling in both approaches: the precision of formal criteria and the sensitivity of content-based interpretation to meaning. Moreover, this use-centered framework supports a unified treatment of theory comparison, expansion, and revision, and sheds light on questions in semantics, logic, and metaphysics.

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